TIPS4RM

Leading MATH Success

TIPS4RM Targeted Implementation and Planning Supports for Revised Mathematics

Mathematical Processes

Sample Adjusted Lessons

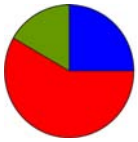
Grade 8

Ontario

Mathematical Process	Grade 8 TIPS4RM Lesson
Reasoning and Proving	Unit 10 Day 3
Reflecting	Unit 8 Day 1
Selecting Tools and Computational Strategies	Unit 5 Day 1
Connecting	Unit 2 Day 2
Representing	Unit 6 Day 2

Unit 10: Day 3: Challenges Are Shaping Up…

Grade 8



Math Learning Goals

- Investigate the relationship of the areas of semi-circles drawn on the sides of a right-angled triangle.

Materials

Assessment Opportunities

Minds On…

Whole Class → Discussion

Collect the Home Activity for assessment.

Using a sketch, reinforce the concept of the Pythagorean relationship. Stress that the relationship is true for right-angled triangles only.

Ask: Does this relationship work with shapes other than squares drawn on the right sides of a right-angled triangle?

N-agon areas.gsp
This GSP®4 sketch can be used to explore or consolidate.

Action!

Pair/Share → Investigation

Using grid paper, students draw a right-angled triangle. They construct semi-circles on the legs and hypotenuse of the triangle and calculate the areas of each semi-circle to determine the relationship the same way they did with squares on Day 2. Students share their work with another pair and explain their reasoning.

Reasoning & Proving/Observation/Checklist: Observe students as they explain their reasoning.

Review how to determine the area of a circle.

Consolidate Debrief

Whole Class → Discussion/Brainstorm

Summarize the findings of their investigation. The sum of the area of the semi-circles on the legs is equal to the area of the semi-circle on the hypotenuse. Pythagorean relationship works for a right-angled triangle using squares and semi-circles drawn on the sides.

Ask:

- What other shapes will work?
- Under what conditions will other shapes work?

Students complete the After column for question 4 of the Anticipation Guide (Day 2 BLM 10.2.1).

Do not answer these questions – this is a brainstorm only.

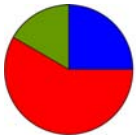
Exploration Practice

Home Activity or Further Classroom Consolidation

Draw a right-angled triangle with the length of legs being whole numbers. On each side of the triangle draw a rectangle (no squares are allowed!). Calculate the areas of the three rectangles. Does this demonstrate the Pythagorean relationship? Explain. Repeat with two more triangles.

Unit 10: Day 3: Challenges Are Shaping Up (A)

Grade 8



Mathematical Process Goals

- Hypothesize and perform multiple trials, and draw conclusions about the relationship among the areas of figures drawn on the sides of a right triangle.

Materials

- GSP®4
- calculators

Assessment Opportunities

Minds On…

Whole Class → Discussion

Using a sketch, reinforce the concept of the Pythagorean relationship with squares drawn on the sides of a right angle triangle. Stress that the relationship is true for right-angled triangles only.

Ask students to make a hypothesis about the relationship of the areas of a figure other than a square drawn on the sides of a right-angled triangle.

Mathematical Process Focus: Reasoning and Proving

See TIPS4RM Mathematical Processes package pp. 3–4.

Action!

Pair/Share → Investigation

Using grid paper, pairs of students draw a right-angled triangle. They construct semi-circles on the legs and hypotenuse of the triangle and calculate the areas of each semi-circle to determine the relationship (the same way they did with squares on TIPS4RM Unit 10 Day 2). Students share their work with another pair and explain their reasoning.

Mathematical Process/Reasoning and Proving/Checklist: Observe students as they explain their reasoning.

Possible guiding questions:

- Can we show that this is true for all cases?
- How can we present an argument in a logical and organized manner?
- What other situations need to be considered?

Whole Class → Discussion

Lead a discussion to facilitate students' understanding that since all pairs have the same result for the investigation, it shows that it works but it doesn't "prove" that it is always true. It does make a convincing argument.

Consolidate Debrief

Whole Class → Discussion/Brainstorm

After students summarize the findings of their investigation, ask:

- What other shapes do you think will work?
- Under what conditions will other shapes work?
- How can you show this or disprove this?

Using GSP®4, demonstrate that regardless of the length of the sides of the right-angled triangle, if the figures drawn on the sides are similar, the sum of the areas of the figures drawn on the two legs is equal to the area of the figure drawn on the hypotenuse.

Students draw a right-angled triangle with the length of legs being whole numbers. On each side of the triangle they draw a rectangle (No squares are allowed!). Calculate the areas of the three rectangles.

Ask: Does this demonstrate the Pythagorean relationship? Explain. Students repeat with two more triangles.

This will demonstrate a "counter-example" and reinforce the need for the figures to be similar.

Home Activity or Further Classroom Consolidation

Concept Practice

Do A or B:

A) Look around your home or neighbourhood and identify where you see right-angled triangles and show the Pythagorean relationship on it, using words, symbols, and diagrams.

B) Considering today's discussion make a hypothesis, about: The relationship of the longest side of any right-angled triangle and its opposite angle. Then try to show that it works or disprove it

Unit 8: Day 1: Size It Up

Grade 8

Math Learning Goals

- Investigate proportional situations using everyday examples.
- Identify proportional and non-proportional situations.

Materials

- relational rods
- measuring tapes
- BLM 8.1.1, 8.1.2, 8.1.3, 8.1.4
- assorted cylinders

Assessment Opportunities

Minds On…

Pairs → Anticipation Guide

Distribute BLM 8.1.1. Students highlight key words in each of the six statements, then complete the Before column of the Anticipation Guide for Proportional Reasoning. Upon completion students explain their reasoning to a partner. Volunteers explaining their reasoning.

See Think Literacy Mathematics: Grades 7–9, Anticipation Guide, p. 10.

Action!

Small Groups → Investigation

Explain the instructions at each station (BLM 8.1.2 and 8.1.4). Students rotate through three of them (or more if time allows). Students will record data on BLM 8.1.3.

Whole Class → Discussion

Compare the data collected at each station. Discuss data that doesn't fit due to incorrect measurements or calculations. Identify proportional and non-proportional situations (BLM 8.1.4).

Communicating/Observation/Mental Note: Observe as students rotate through the stations. Note any potential misunderstandings. These can be addressed in **Consolidate Debrief.**

See Think Literacy Mathematics: Grades 7–9, p. 38.

Consolidate Debrief

Whole Class → Discussion

Groups discuss their findings for each station.

Complete and post a class Frayer model for the word Proportion (BLM 8.1.4).

Students revisit their original responses on the anticipation guide and complete the After column.

Concept Practice

Home Activity or Further Classroom Consolidation

Find some examples of proportional situations at home and add them to the Frayer model.

Unit 8: Day 1: Size It Up (A)

Grade 8

Mathematical Process Goals

- Reflect on findings of investigations.
- Reflect on new learning of proportional relationships.

Materials

- relational rods
- measuring tapes
- assorted cylinders

Assessment Opportunities

Minds On…

Pairs → Anticipation Guide

Distribute TIPS4RM BLM 8.1.1. Students highlight key words in each of the six statements, then complete the Before column of the Anticipation Guide for Proportional Reasoning. Upon completion students explain their reasoning to a partner. Volunteers explaining their reasoning to the class.

Mathematical Process Focus: Reflecting

See TIPS4RM Mathematical Processes package p. 5

Action!

Small Groups → Investigation

Explain the instructions at each station (TIPS4RM BLM 8.1.2 and 8.1.4). Students rotate through three stations or more if time allows. Students record data on TIPS4RM BLM 8.1.3 and reflect on their findings by answering the questions provided.

Possible guiding questions:

- Does the answer seem reasonable?
- How does this compare to …?

Mathematical Process/Reflecting/Checklist: Observe students as they rotate through the stations. Note their reflections. Share these in **Consolidate and Debrief**.

Whole Class → Discussion

Groups discuss their findings for each station, emphasize the reflection process. Pose questions such as:

- Why do you think the data in this situation did not fit well?
- How did you know if the situation was proportional or non-proportional?

Consolidate Debrief

Whole Class → Discussion

As groups discuss their findings, they reflect on how they knew…

Students revisit their original responses on the Anticipation Guide and complete the After column.

Students reflect on why their responses have changed or not.

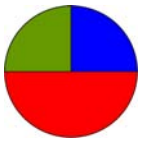
Concept Practice

Home Activity or Further Classroom Consolidation

Find some examples of proportional situations at home and add them to the Frayer model.

Unit 5: Day 1: Pizza and Cake

Grade 8



Math Learning Goals

- Activate and assess prior knowledge of fractions.
- Reason about fractions from a variety of perspectives and representations.

Materials

- geoboards
- fraction circles
- chart paper
- markers
- BLM 5.1.1, 5.1.2

Assessment Opportunities

Minds On.

Small Groups → Exploration/Presentation

Distribute fraction cards (BLM 5.1.1). Explain the task, including the presentation. Tell them that they are to use a variety of strategies and tools, including estimation, manipulatives, diagrams, anchors (of 0, $\frac{1}{2}$, 1), and equivalent forms (decimals, percents) to complete the task and include in their presentation.

Students find other students who have cards of the same colour, arrange their group's fractions in order, and discuss their reasoning.

Two groups form a larger group to discuss the strategies and tools they used and plan and make a presentation.

Curriculum Expectations/Observation/Anecdotal Notes: Observe students' comfort and facility with fractions to determine what fraction experiences are needed in this unit.

Students may use a calculator to change each fraction to a decimal.

Students use their knowledge of multiples to determine common denominators.

Action!

Small Groups → Modelling

Set up multiple stations with the two activities (BLM 5.1.2).

Students work at one of the stations for half the time, then switch stations. They prepare their solutions on chart paper for a whole-class discussion.

Students review the factors of composite numbers, as they reduce fractions.

Consolidate Debrief

Whole Class → Discussion

Use the chart paper solutions to consolidate understanding:

- Equal fraction pieces (same area) can have different shapes.
- Equal fractions can be expressed in different ways.
- Fractions can be expressed with common denominators for addition.
- $\frac{n}{n} = 1$
- Fractions can be reduced when numerator and denominator share a common factor greater than 1.

Discuss how to use common denominators and benchmarks (0, $\frac{1}{2}$, and 1) when comparing fractions.

See *Think Literacy: Mathematics* pp. 76–81, Graphic Organizers.

Home Activity or Further Classroom Consolidation

Make a mind map of things you remember about fractions. Include:

- terminology, e.g., *proper*, *improper*
- how to add and subtract fractions using symbols
- how to represent fractions on a number line

Reflection

Unit 5: Day 1: Pizza and Cake (A)

Grade 8

Mathematical Process Goals

- Select tools to represent fractions.
- Justify choice of tool.
- Explain computational strategies used to compare fractions.

Materials

- variety of tools for fractions
- chart paper
- markers

Assessment Opportunities

Minds On…

Small Groups → Exploration

Each group selects one of the fractions $\left(\frac{6}{10}, \frac{3}{4}, 1\frac{1}{2}, \frac{2}{3}\right)$ and represents it in as many ways as they can using the tools provided.

Whole Class → Gallery Walk

Students look at all the different ways the tools were used to represent the various fractions.

Mathematical Process Focus: Selecting Tools and Computational Strategies

See TIPS4RM Mathematical Processes package pp. 6–7.

Action!

Small Groups → Exploration

Distribute fraction cards (TIPS4RM BLM 5.1.1). Explain the task, including the presentation. Tell students that they are to use a variety of strategies and tools, including estimation, manipulatives, diagrams, benchmarks (of 0, $\frac{1}{2}$, 1), and equivalent forms (decimals, percents) to complete the task and include in their presentation.

Students find other students who have cards of the same colour, arrange their group's fractions in order, and discuss their reasoning.

Each group prepares a presentation to share with the class the strategies and tools they used to complete the task.

Possible guiding questions:

- What other tools could you use?
- Explain why you chose not to use them.
- Explain why you chose this computational strategy.
- Explain another computational strategy that could be used to solve this problem.

Mathematical Process/Selecting Tools and Computational Strategies/ Checklist: Observe students for appropriateness and effectiveness of their choice of tool and/or computational strategy they used to complete the task.

A variety tools that can be used to represent fractions: fraction tiles, fraction strips, fraction circles, square and circle geoboards, relational rods, linking cubes, and links.

Have calculators available for support.

Consolidate Debrief

Whole Class → Presentations

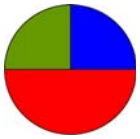
Each group presents to the class.

Determine if students need further fraction experiences in this unit.

Home Activity or Further Classroom Consolidation

Application

Find a recipe that requires the use of fractions and bring it to class next day.



Mathematical Process Goals

- Selects tools to model fractions.

Materials

- fraction circles
- geoboards

Assessment Opportunities

Minds On…

Small Groups → Modelling

Each group chooses one of the recipes from the Home Activity and selects tools to model the ingredients that the recipe requires. Different tools can be used to model the different types of quantities, e.g., teaspoon, tablespoon, cup.

Mathematical Process Focus: Selecting Tools and Computational Strategies

See TIPS4RM Mathematical Processes package pp. 6–7.

Action!

Small Groups → Modelling

Set up multiple stations with the two activities (TIPS4RM BLM 5.1.2).

Students work at one of the stations for half the time, then switch stations. They prepare their solutions on chart paper for a whole-class discussion.

Consolidate Debrief

Whole Class → Discussion

Use the chart-paper solutions to consolidate understanding:

- Equal fraction pieces (same area) can have different shapes.
- Equal fractions can be expressed in different ways.
- Fractions can be expressed with common denominators for addition.
- $\frac{n}{n} = 1$
- Fractions can be reduced when numerator and denominator share a common factor greater than 1.

Discuss how to use common denominators and benchmarks ($0, \frac{1}{2}$, and 1) when comparing fractions.

Home Activity or Further Classroom Consolidation

Make a mind map of things you remember about fractions. Include:

- terminology, e.g., *proper*, *improper;*
- how to add and subtract fractions using symbols;
- how to represent fractions on a number line.

Reflection

Unit 2: Day 2: Different Representations of the Same Pattern

Grade 8

Math Learning Goals

- Examine (linear) patterns involving whole numbers presented in a variety of forms, e.g., as a numerical sequence, a graph, a chart, a physical model, in order to develop strategies for identifying patterns.

Materials

- a visual pattern
- BLM 2.2.1, 2.2.2, 2.2.3
- linking cubes
- rulers

Assessment Opportunities

Minds On…

Pair/Share → Patterning

Model how to share a visual pattern, e.g., art, nautilus shell, in both words and pictures. Student A shares the pattern in words and pictures with Student B. Student B shares the pattern in words and pictures with Student A. Regroup pairs to form groups of four.

Student A in each pair will share Student B's pattern with the group. Student B in each pair will share Student A's pattern with the group.

Interesting visual patterns can be found by doing an online image search.

Action!

Small Groups → Investigation

In heterogeneous groups, students rotate through the stations (BLM 2.2.1) They record their work on BLM 2.2.2. (The empty circle area on this BLM is used on Day 3.)

Whole Class → Connecting

Students share their findings and record any corrections on their worksheet. They label the four rectangular sections as: Numerical Model, Graphical Model, Patterning Rule, Concrete Model (BLM 2.2.2).

Lead students to the conclusion that all of these representations show the same pattern:

- What do you notice about the table of values and the concrete representation?
- What are the similarities? (i.e., they are all representations of the same pattern)

Curriculum Expectations/Observation/Checklist: Circulate to assess understanding that the representations all show the same pattern.

Consolidate Debrief

Whole Class → Four Corners

Post charts in the four corners of the room labelled as: Graphical Model, Patterning Rule, Concrete Model, Numerical Model. Below each label, draw a rough diagram to aid visual learners.

Pose the question: For which model did you find it easiest to extend the pattern?

Students travel to the corner that represents their answer and discuss why they think that they found that method easier. One person from each corner shares the group's findings.

Home Activity or Further Classroom Consolidation

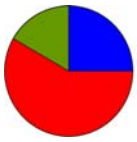
Practice

Complete the practice questions.

Provide students with appropriate practice questions showing multiple ways of representing linear patterns.

Unit 2: Day 2: Different Representations of the Same Pattern (A)

Grade 8



Mathematical Process Goals

- Make connections between new and prior knowledge of patterns to represent them in a variety of forms.
- Make connections between different representations of a pattern, i.e., numeric, graphical, concrete, and algebraic.

Materials

- BLM 2.2.4(A), 2.2.5(A), 2.2.6(A)
- pattern blocks
- linking cubes
- rulers

Assessment Opportunities

Minds On…

Individual → Anticipation Guide

Students record a response for each statement in the Anticipation Guide in the Before column (BLM 2.2.4(A)). [This will activate students' prior learning and experience with patterns as well as help them make personal connections with the topic of patterning.]

Mathematical Process Focus: Connecting

See TIPS4RM Mathematical Processes package p. 8

Pairs/Share → Patterning

Model how to share a visual pattern, e.g., art, nautilus shell, in both words and pictures. Student A shares the pattern in words and pictures with Student B. Student B shares the pattern in words and pictures with Student A.

Students complete a patterning activity, using visual pattern samples BLM 2.2.5(A).

Action!

Small Groups → Investigation

In heterogeneous groups, students rotate through the stations (TIPS4RM BLM 2.2.1) They record their work on TIPS4RM BLM 2.2.2. (The empty circle area on this BLM is used on Day 3.)

Word Wall:
- numerical model
- graphical model
- patterning rule
- concrete model

Whole Class → Connecting

Help students make connections and consolidate their understanding of different representations of the same pattern.

Possible guiding questions:
- Describe the connections you see among the representations.
- How can you relate your understanding of different representations of the same pattern to help you complete the Find the Missing Representation! activity (BLM 2.2.6(A))?

Mathematical Process/Connecting/Checklist: Observe how the students communicate about how the concepts are connected.

Consolidate Debrief

Individual → Anticipation Guide

Students record a response to each statement in the Anticipation Guide in the After column. They compare the Before and After columns and discuss any changes of opinion explaining why they changed their mind.

Whole Class → Four Corners → Connecting

Use the statements in the Anticipation Guide to reflect on the stations activity.

Create a Venn diagram to summarize the similarities and differences between the four representations.

Concept Practice

Home Activity or Further Classroom Consolidation

Complete the practice questions.

Complete Find the Missing Representation! (worksheet 2.2.6 (A)).

Provide students with appropriate practice questions showing multiple ways of representing linear patterns.

2.2.4(A): Anticipation Guide – Representing Patterns

Instructions

- Check Agree or Disagree beside each statement below *before* you start rotating through the stations.
- Compare your choice and explanation with a partner.
- Revisit your choices at the end of the investigation. Compare the choices that you make *after* the investigation with the ones that you made before the investigation.

Anticipation Guide

Before		Statement	After	
Agree	Disagree		Agree	Disagree
		1. Patterns that are seen in daily life aren't really mathematical.		
		2. The only way to represent a pattern is by using a chart or table of values.		
		3. You don't need to be flexible in your thinking in order to identify patterns, just follow a few simple rules.		

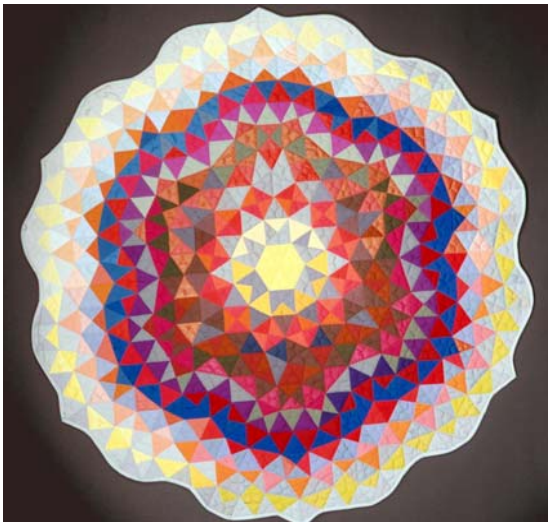
2.2.5(A): Visual Patterns

Quilt

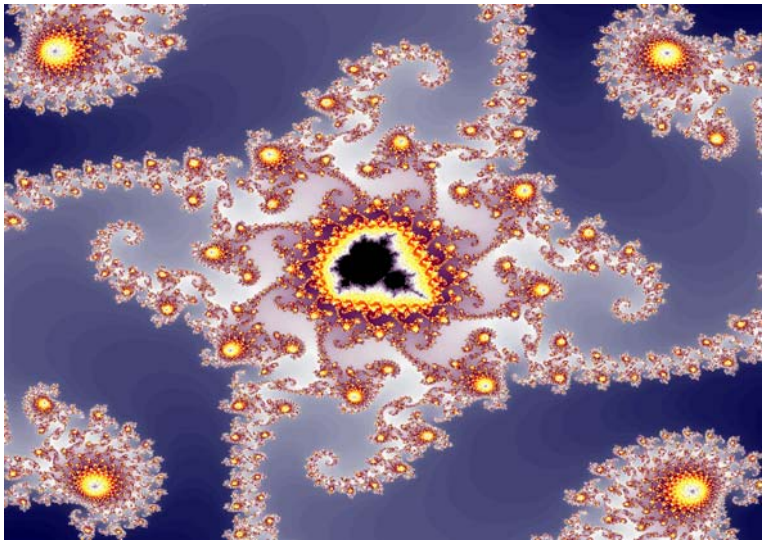
Caucus, 1900–1930

(used with permission from The Textile Museum of Canada)

Round Work
Pieced Cotton Quilt
Judith Tinkl 2002
(used with permission)



Fractal Images
(used with permission)



http://spanky.triumf.ca/pub/fractals/images/MANDEL/2B4A.PNG

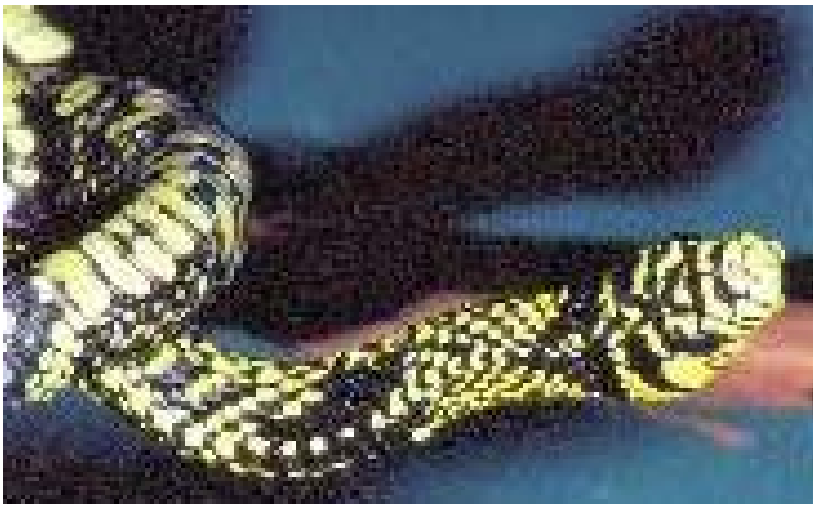
2.2.5(A): Visual Patterns (continued)

Sensitive Fern

Oak Fern

Ostrich Fern

http://www.ontariowildflower.com/fern.htm#groundcedar



Tiger Rat Snake

http://www.torontozoo.com/Animals/details.asp?AnimalId=489

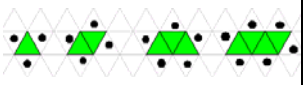
2.2.6(A): Find the Missing Representation

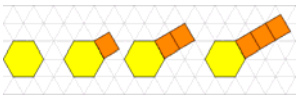
Determine which of the four representations of the pattern is missing – numerical model, graphical model, patterning rule, or concrete model.

Create a model and write the patterning rule for the missing representation.

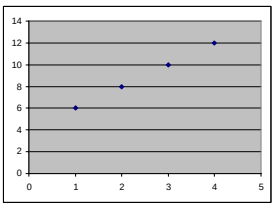
Term	Value
1	3
2	4
3	5
4	6
5	
6	

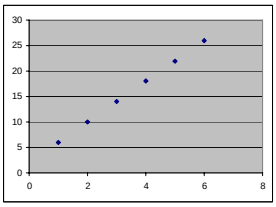
Start with 1 triangular table. Three people can sit at this table. For every 1 additional table connected, the number of people seated increases by 1.



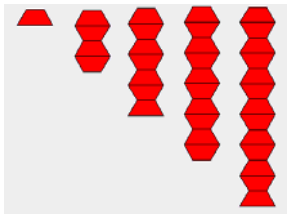


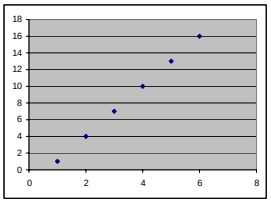
Start with 1 regular hexagon stone. The perimeter is 6. For every 1 square that you add to create the stone path, the perimeter increases by 2.





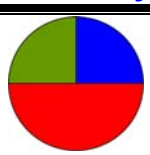
Start with 1 cube. The surface area for 1 cube is 6. For every 1 cube that you add, the surface area increases by 4.





Unit 6: Day 2: Experimental and Theoretical Probability: Part 1

Grade 8



Math Learning Goals

- Represent probability in multiple ways.
- Introduce concepts of theoretical and experimental probability.

Materials

- coins
- BLM 6.2.1, 6.2.2

Assessment Opportunities

Minds On…

Whole Class → Guided Review

Curriculum Expectations/Journal: Collect and assess math journal entry.

Review the meaning of the vocabulary associated with probability situations (BLM 6.2.1). Students brainstorm, write, and share their own statements, using correct terminology. In discussion, focus on those events which students identify as "maybe" to decide whether these events are likely or unlikely to occur. Students explain their reasoning.

Probability is the mathematics of chance.

The probability of an event is a number between 0 and 1; an impossible event, 0; and an event that is certain, 1.

Action!

Pairs → Investigation

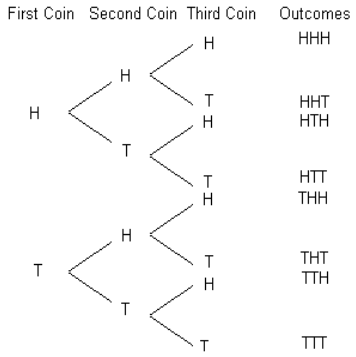
Students toss one coin and state the number of possible outcomes. They toss two coins and suggest possible outcomes.

Demonstrate how a tree diagram can be used to organize the outcomes of their tosses. Point out that the branches represent their choices.

Each pair of students creates a tree diagram for tossing three coins. As an example, when tossing three coins, we wish to see 1 head and 2 tails. What is the probability of this occurring?

Explain that a preference is considered to be a favourable outcome; and the probability of that event is the ratio of the number of favourable outcomes to the total number of possible outcomes.

$$P = \frac{Number\ of\ favourable\ outcomes}{Number\ of\ possible\ outcomes}$$



Each pair tosses two coins twenty times (20 is the sample size) and records each outcome.

They compare their experimental results to the theoretical results. Discuss how changing sample size (to more or fewer than 20) would affect experimental results.

Theoretical probability applies only to situations that can be modelled by mathematically fair objects.

The coin toss provides experimental results.

Experimental probability is based on the results of an experiment and are relative frequencies, giving an estimate of the likelihood that a particular event will occur.

Experimental probabilities are often close to the theoretical probabilities especially if the sample size is large.

Experimental results: $\frac{\#TT}{20}$ $\frac{\#HH}{20}$ $\frac{\#TH\ or\ HT}{20}$ compared to theoretical results

P (TT) = $\frac{1}{4}$ P (one of each) = $\frac{2}{4}$ P (HH) = $\frac{1}{4}$

Students prepare a presentation of their findings.

Consolidate Debrief

Whole Class → Presentation

One student from each pair presents their results for tossing two coins twenty times. Combine whole class data to share results with the larger sample size.

Discuss the effect of sample size on experimental outcomes. Discuss what a probability of 0 and a probability of 1 would mean in the context of coin tosses.

Curriculum Expectations/Presentations/Class Response: Assess communication skills during the student presentation.

Home Activity or Further Classroom Consolidation

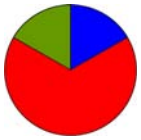
Complete worksheet 6.2.2.

Devise your own simulations using spinners, or a combination of coins and spinners, etc.

Reflection
Concept Practice
Skill Drill

Unit 6: Day 2: Experimental and Theoretical Probability: Part 1 (A)

Grade 8



Mathematical Process Goals
- Represent outcomes in a variety of ways
- Represent probability as a fraction, decimal, percent.

Materials
- coins
- BLM 6.2.1(A)

Assessment Opportunities

Minds On…

Whole Class → Sharing
Students share their findings from Day 1 Home Activity.

Individual → Exercise
Students indicate the Type of Event on BLM 6.2.1(A).

Mathematical Process Focus: Representing

See TIPS4RM Mathematical Processes package p. 9.

Action!

Small Groups → Investigation and Presentation
Students create a menu consisting of 3 appetizers, 4 main courses, and 2 desserts. They determine all the different possible combinations of their menu if a person orders 1 appetizer, 1 main course, and 1 dessert.

Mathematical Process/Representing/Checklist: Observe students for the effectiveness of the representation they used to display the combinations.

Groups share their menus and all possible combinations. Highlight the various representations the students used. Model using a tree diagram, if that representation was not shared.

Whole Class → Guided Instruction
Explain that a preference is considered to be a favourable outcome; the probability of that event is the fraction of the number of favourable outcomes compared to the total number of possible outcomes.

Determine the probability of some favourable events dealing with the menu. Represent the probabilities as fractions, decimals, and percents. Discuss what a probability of 0 and a probability of 1 would mean in the context of the menu.

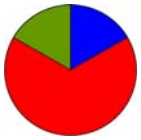
Consolidate Debrief

Small Groups → Investigation
As a group, students share their answers to the Type of Event on BLM 6.2.1(A). Students may adjust their answer, if they wish. As a group, they estimate a value of the probability for each of the events and enter on BLM 6.2.1(A). Students select a fraction, decimal, or a percent to represent the probability for each event.

Possible guiding questions:
- Explain why you chose this representation.
- How does a tree diagram help us to organize the outcomes?
- In which situations did you use a fraction to represent the probability?
- In which situations did you use a decimal to represent the probability?
- In which situations did you use a percent to represent the probability?

Application

Home Activity or Further Classroom Consolidation
Find a newspaper or magazine article that uses probability and bring it to class.



Mathematical Process Goals
- Represent outcomes in a variety of ways.
- Represent probability as a fraction, decimal, percent.

Materials
- coins
- BLM 6.2.2(A)

Assessment Opportunities

Minds On…

Small Group→ Sharing
Students share their newspaper or magazine article and highlight the probability event. They share how the probability was expressed (as a fraction, decimal, or percent).

Whole Class → Discussion
Facilitate a discussion around the articles to introduce the terminology of *experimental* versus *theoretical* probability.

Mathematical Process Focus: Representing

See TIPS4RM Mathematical Processes package p. 9

Action!

Pairs → Investigation
Students toss one coin and state the number of possible outcomes. They toss two coins and suggest possible outcomes.

Each pair tosses two coins twenty times (20 is the sample size) and records each outcome.

Students determine from their sample of 20 trials what the experimental probability is for each event. Express each probability in fraction, decimal and percent form.

Each pair adds their outcomes to the class chart resulting in a larger sample size. Express each event using the larger sample size.

Discuss the theoretical probability of tossing two coins for each of the different events (no tails, 1 tails, 2 tails).

Consolidate Debrief

Whole Class → Guided Discussion
Compare the experimental probability of the large sample to the smaller sample.

Discuss the effect of sample size on experimental outcomes: the larger the sample size the closer the experimental probability will be to the theoretical probability.

Home Activity or Further Classroom Consolidation
Complete worksheet 6.2.2(A).

Reflection
Concept Practice

See BLM 6.2.3 (A) for answers.

6.2.1(A): Talking Mathematically

Name:

Read each statement carefully. Choose from the terms to describe each event and record your answer in the space provided:

- certain or sure
- impossible
- likely or probable
- unlikely or improbable
- maybe
- uncertain or unsure

Event	Type of Event	Probability of Event
1. A flipped coin will show tails.		
2. I will be in school tomorrow.		
3. It will not get dark tonight.		
4. I will have pizza for dinner tonight.		
5. I roll a 3 using a number cube.		
6. It will snow in July.		
7. The teacher will write on the board today.		
8. January will be cold in Ontario.		
9. My dog will bark.		
10. I will get Level 4 on my science fair project.		

6.2.2(A): Investigating Probability

Name:
Date:

Solve the following problems in your notebook:
Represent the probabilities as a fraction, decimal, and a percent.

1. Keisha's basketball team must decide on a new uniform. The team has a choice of black shorts or gold shorts and a black, white, or gold shirt.
   Show the team's uniform choices.
   a) What is the probability the uniform will have black shorts?
   b) What is the probability the shirt will not be gold?
   c) What is the probability the uniform will have the same-coloured shorts and shirt?
   d) What is the probability the uniform will have different-coloured shorts and shirt?

2. Brit goes out for lunch to the local submarine sandwich shop. He can choose white or whole wheat bread, and a filling of turkey, ham, veggies, roast beef, or salami.
   Show all Brit's possible sandwich choices.
   a) How many sub choices are there?
   b) He may also choose a single topping of tomatoes, cheese, or lettuce. Now, how many possible sub choices does he have?
   c) If each possibility has an equal chance of selection, what is the probability that Brit will choose a whole wheat turkey sub topped with tomatoes?
   d) What is the probability of choosing a veggie sub topped with cheese?
   e) What is the probability of choosing a meat sub topped with lettuce on white bread?
   f) What is the probability of choosing a meat sub topped with lettuce?

3. The faces of a cube are labelled 1, 2, 3, 4, 5, and 6. The cube is rolled once.
   List the favourable outcomes for each.
   a) What is the probability that the number on the top of the cube will be odd?
   b) What is the probability that the number on the top of the cube will be greater than 5?
   c) What is the probability that the number on the top of the cube will be a multiple of 3?
   d) What is the probability that the number on the top of the cube will be less than 1?
   e) What is the probability that the number on the top of the cube will be a factor of 36?
   f) What is the probability that the number on the top of the cube will be a multiple of 2 and 3?

6.2.3(A): Investigating Probability (Answers)

Question 1

a) The probability the uniform will have black shorts is $\frac{3}{6} = \frac{1}{2} = 0.5 = 50\%$.

b) The probability the shirt will not be gold is $\frac{4}{6} = \frac{2}{3} = 0.\dot{6} = 66.7\%$.

c) The probability the uniform will have the same-coloured shorts and shirt is $\frac{2}{6} = \frac{1}{3} = 0.\dot{3} = 33.3\%$.

d) The probability the uniform will have different-coloured shorts and shirt is $\frac{4}{6} = \frac{2}{3} = 0.\dot{6} = 66.7\%$.

Question 2

a) Brit has the choice of 2 breads and 5 fillings. So, he has the choice of $2 \times 5 = 10$ sandwiches.
This can be shown using a tree diagram that first has 2 branches (one for each of the bread types) and then 5 branches at the end of the first branches (one for each of the fillings). This will give 10 ends to the tree.

b) You can add 3 branches at the end of each branch to indicate each of 3 topping choices. This gives 30 possible outcomes.

c) Only one of these outcomes is a whole-wheat turkey sandwich topped with tomatoes. So the probability that he chooses this sandwich is $\frac{1}{30} = 0.0\dot{3} = 3.3\%$. It is only one of 30 possible sandwiches.

d) The probability of choosing any veggie sub topped with cheese is $\frac{2}{30} = \frac{1}{15} = 0.0\dot{6} = 6.7\%$.
(The student must remember to use both the whole wheat and white bread possibility in this answer.)

e) The probability of choosing a meat sub topped with lettuce on white bread is $\frac{4}{30} = \frac{2}{15} = 0.1\dot{3} = 13.3\%$.
(The student must remember to use all possible meat selections for this answer.)

f) The probability of choosing a meat sub topped with lettuce is $\frac{8}{30} = \frac{4}{15} = 0.2\dot{6} = 2.7\%$. The student must remember to use all possible meat selections in this answer, and both types of bun.

6.2.3(A): Investigating Probability (Answers) (continued)

Question 3

a) There are 3 odd numbers, so the probability is $\frac{3}{6} = \frac{1}{2} = 0.5 = 50\%$.

b) There is only one number greater than 5, so the probability is $\frac{1}{6} = 0.1\dot{6} = 16.7\%$.

c) There are two multiples of 3, i.e., 3 and 6, so the probability is $\frac{2}{6} = \frac{1}{3} = 0.\dot{3} = 33.3\%$.

d) There is no number less than one, so the probability is zero.

e) There are 5 numbers that are factors of 36, i.e., 1, 2, 3, 4, and 6, so the probability is $\frac{5}{6} = 0.8\dot{3} = 83.3\%$.

f) There is only one number that is a multiple of both 2 and 3, i.e., 6, so the probability is $\frac{1}{6} = 0.1\dot{6} = 16.7\%$.